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SINGLETONS AND SUPERSTRINGS^{*}

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A B S T R A C T

We show that the type IIA, IIB and heterotic superstring actions can be interpreted as the $(8_c, 8_s)$, $(8_c, 8_c)$ and $(8_c, 0)$ superconformally invariant two-dimensional singleton field theories of the three-dimensional anti-de Sitter supergroups $OSp(8/2, \mathbb{R})_c \otimes OSp(8/2, \mathbb{R})_s$, $OSp(8/2, \mathbb{R})_c \otimes OSp(8/2, \mathbb{R})_c$ and $OSp(8/2, \mathbb{R})_c \otimes Sp(2, \mathbb{R})$, respectively.

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In any Lagrangian field theory invariant under the Poincaré group, the covariant fields describing particles of a given spin s transform as a finite dimensional non-unitary representation of the Lorentz group. However, the particle states created by the operator Fourier modes of these fields form the basis of a unitary representation of the Poincaré group. Similarly the Fourier modes of a field in four-dimensional anti-de Sitter space (adS_4) form the basis of a unitary representation of the adS_4 group $SO(3,2)$. In 1963, Dirac¹⁾ discovered two remarkable unitary irreducible representations (UIR) of the adS_4 group $SO(3,2)$ whose Poincaré limit is singular. These representations are referred to as singletons²⁾ and have no Poincaré analogues and no obvious field theoretic interpretation. One way to understand them is in terms of extreme gauge fields whose gauge invariance is such as to allow them to be gauged away everywhere, except at the boundary of adS space³⁾. This interpretation is supported by the analysis of a number of Kaluza-Klein supergravity theories for which the (supersymmetric) ground state is of the form $adS_n \times S^m$. For example, the modes in adS_4 that result from a compactification of $d = 11$ supergravity on S^7 can be grouped into unitary irreps (= irreducible representations) of the adS supergroup $OSp(8/4, \mathbb{R})$ ^{4), 5)}. The singleton irreps^{5), 6)} of this supergroup indeed appear in the harmonic expansion on S^7 of the $d = 11$ fields^{7), 5)}, but can be gauged away. This state of affairs is repeated for the S^4 compactification of $d = 11$ supergravity to $d = 7$ ⁸⁾, where the relevant adS supergroup is $OSp(8^*/4) \cong OSp(6, 2/4)$, and for the S^5 compactification of chiral $N = 2$, $d = 10$ supergravity to $d = 5$, where the relevant adS supergroup is $SU(2, 2/4)$ ⁹⁾. In each case the fields of the singleton supermultiplet that can be gauged away are precisely those of the maximally supersymmetric and conformally invariant matter (spins < 1) field theory of one dimension lower^{*}). Thus, the relevant singleton irrep of $OSp(8/4, \mathbb{R})$ consists of the fields of a $d = 3$, $N = 8$ multiplet with spins $(0, \frac{1}{2})$, while the "singleton" irrep of $OSp(8^*/4)$ consists of the fields of the $d = 6$, $N = 4$ antisymmetric tensor multiplet, and the "singleton" irrep of $SU(2, 2/4)$ consists of the fields of the $d = 4$, $N = 4$ super Yang-Mills multiplet. This result is not surprising since the adS_d group acts as the conformal group in $(d-1)$ -dimensional Minkowski space. The purpose of this letter is to show that the same interpretation can be given to the singleton supermultiplets of the $d = 3$ adS supergroups. In a recent work¹⁰⁾ three of us have classified these

*) Actually, the corresponding supermultiplets in $d = 5$ and 7 are referred to as doubleton supermultiplets for reasons explained in 5) and the last reference in 8). In this paper, we shall refer to all these supermultiplets generically as singleton supermultiplets.

supergroups and constructed the positive energy (lowest weight) UIR's of a large class of these supergroups. The novel feature that arises for $d = 3$ is that the adS_3 group $\text{SO}(2,2)$ is not simple and factorizes as

$$\text{SO}(2,2) \cong \text{SO}(2,1) \otimes \text{SO}(2,1) \cong \text{Sp}(2, \mathbb{R}) \otimes \text{Sp}(2, \mathbb{R}).$$

Thus the extension to an adS_3 supergroup can be done in a variety of ways and one ends up with a rich class of adS supergroups¹⁰⁾. Of interest here are those adS_3 supergroups for which one or both $\text{Sp}(2, \mathbb{R})$ factors is extended to $\text{OSp}(2N/2, \mathbb{R})$ because the supergroups of this form have singleton irreps. In particular, we shall focus on those cases for which one or the other factor is the supergroup $\text{OSp}(8/2, \mathbb{R})$.

The "positive energy" (lowest weight) unitary irreps of $\text{OSp}(2N/2, \mathbb{R})$ decompose into infinite dimensional irreps of $\text{Sp}(2, \mathbb{R})$, transforming according to some finite dimensional irreps of $\text{SO}(2N)$. The irreps of $\text{Sp}(2, \mathbb{R})$ that occur are all of the lowest weight type and can be labelled by the $U(1) \subset \text{Sp}(2, \mathbb{R})$ quantum number l_0 of the lowest weight vector. There are two singleton irreps of $\text{OSp}(2N/2, \mathbb{R})$ and they decompose under the even subgroup $\text{SO}(2N) \otimes \text{Sp}(2, \mathbb{R})$ as follows¹⁰⁾

$$S^1 = [(00 \dots 10), l_0 = \frac{1}{4}] \oplus [(00 \dots 01), l_0 = \frac{3}{4}] \quad , \quad (1)$$

$$S^2 = [(00 \dots 01), l_0 = \frac{1}{4}] \oplus [(00 \dots 10), l_0 = \frac{3}{4}] \quad , \quad (2)$$

where the Dynkin labels $(00 \dots 01)$ and $(00 \dots 10)$ refer to the two half-spin representations of $\text{SO}(2N)$. The odd generators of $\text{OSp}(2N/2, \mathbb{R})$ transform as the finite dimensional $(2N, 2)$ representation of $\text{SO}(2N) \otimes \text{Sp}(2, \mathbb{R})$. Acting on the singleton supermultiplet they interpolate between the two UIR's of $\text{Sp}(2, \mathbb{R})$ transforming in the two half-spin representations of $\text{SO}(2N)$. For $\text{SO}(8)$ the half-spin representations are eight-dimensional just as the vector representation. Thus the singleton supermultiplets of $\text{OSp}(8/2, \mathbb{R})$ have the decomposition

$$S^1 = (8_s, l_0 = \frac{1}{4}) \oplus (8_c, l_0 = \frac{3}{4}) \quad , \quad (3)$$

$$S^2 = (8_c, l_0 = \frac{1}{4}) \oplus (8_s, l_0 = \frac{3}{4}) \quad , \quad (4)$$

where 8_v and 8_c are the two half-spin representations of $SO(8)$. Now the representations of $SO(8)$ exhibit the unique triality symmetry which does not extend to general $SO(2N)$ ($N \neq 4$). This is a reflection of the cyclic symmetry of the $SO(8)$ Dynkin diagram. The principle of triality allows us to define three different forms of $OSp(8/2, \mathbb{R})$ [in fact, of all $OSp(8/2M, \mathbb{R})$]. These different $OSp(8/2, \mathbb{R})$ superalgebras are distinguished by the transformation properties of the odd generators under the even subgroup $SO(8) \otimes Sp(2, \mathbb{R})$:

$$OSp(8/2, \mathbb{R})_v = [(28, 1) + (1, 3)] + [(8_v, 2)] \quad , \quad (5)$$

$$OSp(8/2, \mathbb{R})_s = [(28, 1) + (1, 3)] + [(8_s, 2)] \quad , \quad (6)$$

$$OSp(8/2, \mathbb{R})_c = [(28, 1) + (1, 3)] + [(8_c, 2)] \quad . \quad (7)$$

The singleton supermultiplets given by (3) and (4) are those of $OSp(8/2, \mathbb{R})_v$. To obtain the singleton supermultiplets of $OSp(8/2, \mathbb{R})_s$ and $OSp(8/2, \mathbb{R})_c$ one needs only to do a triality permutation of the eight-dimensional representations 8_v , 8_s and 8_c . Thus for $OSp(8/2, \mathbb{R})_s$ we have the following $SO(8) \otimes Sp(2, \mathbb{R})$ decomposition of the singleton supermultiplets

$$S_s^1 = (8_v, l_0 = \frac{1}{4}) \oplus (8_c, l_0 = \frac{3}{4}) \quad , \quad (8)$$

$$S_s^2 = (8_c, l_0 = \frac{1}{4}) \oplus (8_v, l_0 = \frac{3}{4}) \quad . \quad (9)$$

Similarly, for the singleton irreps of $OSp(8/2, \mathbb{R})_c$ we have

$$S_c^1 = (8_s, l_0 = \frac{1}{4}) \oplus (8_v, l_0 = \frac{3}{4}) \quad , \quad (10)$$

$$S_c^2 = (8_v, l_0 = \frac{1}{4}) \oplus (8_s, l_0 = \frac{3}{4}) \quad . \quad (11)$$

Interpreting $Sp(2, \mathbb{R}) \cong SO(2, 1)$ as the one-dimensional conformal group we can associate with the UIR's of $Sp(2, \mathbb{R})$ labelled by $l_0 = 1/4$ and $l_0 = 3/4$ a one-dimensional "bosonic" field $X(\xi)$ and a "fermionic" field $\theta(\xi)$, respectively. Thus to the singleton supermultiplets of $OSp(8/2, \mathbb{R})$ given above we can associate a supermultiplet of fields (X, θ) with different $SO(8)$ labels. Of particular interest to us are the supermultiplets for which the "bosonic" fields $X(\xi)$

transform in the vector representation of $SO(8)$. This is because we will eventually identify these fields with the transverse co-ordinates in a ten-dimensional space-time. Thus the relevant supermultiplet of fields are

$$(X^i(\zeta), \Theta^\alpha(\zeta)) \quad (12)$$

corresponding to the singleton irrep S_s^1 and the multiplet

$$(X^i(\zeta), \Theta^\alpha(\zeta)) \quad (13)$$

corresponding to the singleton irrep S_c^2 . [The indices i, j are 8_v indices; α, β, \dots are 8_s indices and $\dot{\alpha}, \dot{\beta}, \dots$ are 8_c indices of $SO(8)$.]

Let us now consider an adS_3 supergroup of the form

$$OSp(8/2, \mathbb{R})_{A+} \otimes OSp(8/2, \mathbb{R})_{B-}, \quad (14)$$

where A, B denote the different forms (v, s, c) of the superalgebra $OSp(8/2, \mathbb{R})$ and $+$ and $-$ refer to the fact that the respective $Sp(2, \mathbb{R})$ factor acts on the light-cone co-ordinates $\xi_+ = \tau + \sigma$ and $\xi_- = \tau - \sigma$, respectively. This is an adS supergroup for which both $Sp(2, \mathbb{R})$ factors have been "supersymmetrized". It corresponds to an $(8_A, 8_B)$ supersymmetry in the sense of Ref. 10). For given indices A and B the superalgebra (14) has four singleton supermultiplets. The singleton supermultiplets of fields that are of interest are those obtained by tensoring the supermultiplets (12) and (13). For the adS_3 supergroup

$$OSp(8/2, \mathbb{R})_{c+} \otimes OSp(8/2, \mathbb{R})_{c-} \quad (15)$$

we have the supermultiplet

$$(X^i(\zeta_+), X^i(\zeta_-); \Theta^\alpha(\zeta_+), \Theta^\alpha(\zeta_-)) \quad (16)$$

For the supergroup

$$OSp(8/2, \mathbb{R})_{c+} \otimes OSp(8/2, \mathbb{R})_{s-} \quad (17)$$

we have the interesting supermultiplet

$$(X^i(\xi_+), X^i(\xi_-); \Theta^\alpha(\xi_+), \Theta^\alpha(\xi_-)) . \quad (18)$$

The field theories of the adS_3 singleton supermultiplets (16) and (18) will be superconformally invariant two-dimensional field theories. Using a triality modified form of the notation of 11), they will have an $(8_c, 8_c)$ and an $(8_c, 8_s)$ conformal supersymmetry. This interpretation is confirmed by a study of $\text{OSp}(8/2, \mathbb{R}) \oplus \text{OSp}(8/2, \mathbb{R})$ as the finite dimensional superconformal group¹⁰⁾. The fields of (16) and (18) are of course only appropriate for an on-shell description of a $d = 2$ field theory. To go off-shell we should replace them by $d = 2$ covariant fields:

$$(X^i(\xi); \Theta_L^\alpha(\xi), \Theta_R^\alpha(\xi)) \quad (19)$$

and

$$(X^i(\xi); \Theta_L^\alpha(\xi), \Theta_R^{\dot{\alpha}}(\xi)) , \quad (20)$$

where $\xi = (\tau, \sigma)$ and $L(R)$ indicates a chiral (anti-chiral) $d = 2$ spinor. It is obvious, at this point, that the desired actions are just those of Green and Schwarz¹²⁾ for closed superstrings of type IIA and IIB in the light-cone gauge. For the latter we have (omitting factors of $4\pi\alpha$, etc.) the action

$$S = \int d^2\xi \left\{ \partial_+ X^i \partial_- X^i + \Theta^\alpha \not{\partial} \Theta^\alpha \right\} . \quad (21)$$

This action has $(8_c, 8_c)$ type $d = 2$ supersymmetry¹²⁾. To see that it has an $(8_c, 8_c)$ type conformal supersymmetry, let us restrict $\text{SO}(8)$ to its $\text{SU}(3)$ subgroup under which both 8_v and 8_s decompose as $3 \oplus \bar{3} \oplus 1 \oplus 1$. Then the action (21) can be interpreted as a conventional supersymmetric σ -model with a flat target space¹³⁾. This is certainly $d = 2$ conformal invariant. But conformal invariance together with $(8_c, 8_c)$ supersymmetry implies $(8_c, 8_c)$ superconformal invariance. It seems likely that this is the unique action with $(8_c, 8_c)$ conformal supersymmetry given the very stringent conditions that are already implied on the target space of a $d = 2$ σ -model by $(4, 4)$ supersymmetry¹⁴⁾.

The heterotic superstring can similarly be thought of as a singleton field theory, but in this case we must consider an adS_3 supergroup of the form

$$OSp(8/2, \mathbb{R})_{c+} \otimes Sp(2, \mathbb{R})_- \quad (22)$$

There are two singleton irreps of $Sp(2, \mathbb{R})$, which are the analogues of the "Di" and the "Rac" representations of adS_4 group $SO(3, 2)^{1), 2)}$. They can be associated with the Fourier modes of a one-dimensional "bosonic" field $\phi(\xi)$ and a "fermionic" field $\psi(\xi)$. To obtain the fields of the heterotic string we must take 24 bosonic fields $\phi^a(\xi_-)$, $a = 1, \dots, 24$ and a singleton supermultiplet of $OSp(8/2, \mathbb{R})_{c+}$, i.e.,

$$(X^i(\xi_+), \phi^a(\xi_-); \theta^\alpha(\xi_+)) \quad (23)$$

These are then the fields of a two-dimensional $(8_c, 0)$ superconformally invariant field theory, the heterotic superstring¹⁵⁾.

It was observed sometime ago that the tensor product of the singleton irreps of the adS_4 group $SO(3, 2)$ decomposes into massless irreps only¹⁶⁾. Using the methods of Ref. 17), one can prove similar results for all non-compact groups and supergroups¹⁸⁾. For example, the tensor product of two singleton supermultiplets of adS supergroups in any dimension decompose into massless supermultiplets only. The tensor product of more than two singleton supermultiplets decompose into massive UIR's of the respective adS supergroup¹⁸⁾. The superstring theories exhibit similar features in a different context. Their spectra consist of massless and massive states of ever increasing mass. For example the massless excitations of the type IIB superstring are found in the tensor product of two singleton supermultiplets, e.g.,

$$(\mathcal{S}_V \oplus \mathcal{S}_S) \otimes (\mathcal{S}_V \oplus \mathcal{S}_S) \quad (24)$$

with the $SO(8)$ irreps being thought of as $d = 10$ "helicity". It is equally the $SO(8)$ content of the $N = 16$, $d = 3$ supergravity¹⁹⁾ with $SO(8)$ considered as an internal symmetry group. Thus, there is a curious parallel between $d = 10$ and $d = 3$ supergravity theories.

We shall conclude this letter with a construction of the singleton irreps of $OSp(8/2, \mathbb{R})_V$. We do this by the oscillator method¹⁷⁾. We introduce the superannihilation and creation operators

$$\eta_A = \begin{pmatrix} a \\ \alpha_\mu \end{pmatrix} ; \quad \eta^A = \begin{pmatrix} a^\dagger \\ \alpha^\mu \end{pmatrix} , \quad (25)$$

where a is bosonic and $\alpha^\mu = \alpha_\mu^\dagger$ ($\mu=1,2,3,4$) are fermionic satisfying the supercommutation relations

$$\{ \eta_A , \eta^B \} = \delta_A^B . \quad (26)$$

The operators η_A and η^A transform in the $(\bar{1}, \bar{4})$ and $(1, 4)$ representations of the maximal compact subsupergroup $U(1/4)$ of $OSp(8/2, \mathbb{R})_V$. With respect to this subsuperalgebra the generators of $OSp(8/2, \mathbb{R})_V$ have a Jordan decomposition:

$$L = L^- \oplus L^0 \oplus L^+ , \quad (27)$$

where $L^0 = U(1/4) = U(1) \otimes SU(1/4)$. The L^+ , L^- and L^0 have the following realization as bilinears in the superoscillators η_A, η^B :

$$\begin{aligned} L^- &= \eta_A \eta_B , \\ L^0 &= \eta^A \eta_B + (-1)^{(\deg A)(\deg B)} \eta_B \eta^A , \\ L^+ &= \eta^A \eta^B . \end{aligned} \quad (28)$$

It is easy to verify that these bilinears generate the Lie superalgebra $OSp(8/2, \mathbb{R})_V$ in a super-Hermitian basis^{(10), (17)}.

Now consider any set of states $|\Omega\rangle$, in the super Fock space of η^A , that transform irreducibly under the maximal compact subsupergroup $U(1/4)$ and are annihilated by the operators belonging to the L^- space. Then by acting on $|\Omega\rangle$ repeatedly by the operators belonging to the L^+ space one generates an infinite set of states that form the basis of a UIR of $OSp(8/2, \mathbb{R})_V$. The set of states $|\Omega\rangle$ is referred to as the lowest weight vector of the respective UIR of $OSp(8/2, \mathbb{R})_V$. The infinite set of states that form the basis of a UIR of $OSp(8/2, \mathbb{R})_V$ can be decomposed into lowest weight UIR's of $Sp(2, \mathbb{R})$ transforming in certain irreps of $SO(8)$. The action of the bilinear operator $a^\dagger a^\dagger$ belonging to L^+ moves one within a UIR of $Sp(2, \mathbb{R})$ while the bilinear operator $\alpha^\mu \alpha^\nu \in L^+$ moves one within an irrep of $SO(8)$. The odd generator $a^\dagger \alpha^\mu \in L^+$ is a supersymmetry generator and moves one from a UIR of $Sp(2, \mathbb{R})$ with a given $SO(8)$ irrep to another UIR of $Sp(2, \mathbb{R})$ with different $SO(8)$ transformation properties.

Since we have realized the generators of $OSp(8/2, \mathbb{R})_V$ as bilinears of a single superoscillator η^A there exist only two lowest weight vectors $|Q\rangle$ transforming irreducibly under $U(1/4)$ and are annihilated by L^- . These lowest weight vectors are the vacuum state $|0\rangle$ and the one-particle state $\eta^A|0\rangle$ which consists of one bosonic and four fermionic degrees of freedom:

$$\eta^A|0\rangle = a^\dagger|0\rangle \oplus \alpha^\mu|0\rangle \quad . \quad (29)$$

The singleton supermultiplet determined by the vacuum state $|0\rangle$ as the lowest weight vector is simply the multiplet S^1 in Eq. (3) while the supermultiplet corresponding to the lowest weight vector $\eta^A|0\rangle$ is the supermultiplet S^2 given in Eq. (4).

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